

Flavour dynamics with general scalar fields^{*}

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Abstract. We consider a spontaneously broken gauge theory based on the standard model (SM) group $G = SU(2) \times U(1)$ with scalar fields that carry arbitrary representations of G , and we investigate some general properties of the charged and neutral current involving these fields. In particular we derive the conditions for having real or complex couplings of the Z boson to two different neutral or charged scalar fields, and for the existence of CP-violating Z -scalar-scalar couplings. Moreover, we study models with the same fermion content as in the SM, with one $SU(2)$ Higgs singlet, and an arbitrary number of Higgs doublets. We show that the structure of the Z -Higgs boson and of the Yukawa couplings in these models can be such that CP-violating $ZbbG$ form factors which conserve chirality are induced at the one-loop level.

1 Introduction

The standard model (SM) of elementary particle physics [1] has been very successful, so far, when compared to experiments. For instance LEP1 and SLC, with its precision data, have proved to be an ideal testing ground of the SM, where the theory, including its quantum corrections, has been checked (for recent reviews, see [2,3]). However, one crucial aspect of the SM has remained practically unexplored experimentally till to date: the electroweak symmetry breaking sector. In the standard picture an elementary scalar field¹ is responsible for spontaneous breaking of the electroweak gauge group $G = SU(2) \times U(1)$ and for the generation of particle masses [4,5]. However, extensions of the SM, for which there are a number of well-known theoretical motivations, almost invariably entail a larger scalar field content than in the SM. That is, additional Higgs fields, but possibly also scalar leptoquarks or, in supersymmetric extensions of the SM, squarks and sleptons.

In this article we shall investigate an $SU(2) \times U(1)$ gauge theory with an arbitrary number of scalar fields. For ease of notation we will collectively denote these fields as Higgs fields. Our aim is to answer some general questions concerning the charged and in particular the neutral current involving these fields, namely:

What are the conditions for having a real or complex coupling of the Z boson to two different neutral or charged physical Higgs fields?

Can there be CP-violating Z -Higgs couplings? What are the conditions that complex phases in such couplings can or cannot be “rotated away”?

Our article is organised as follows. In Sect. 2 we introduce the general Higgs field and discuss spontaneous symmetry breaking. In Sect. 3 we study the question of non-diagonal Z -Higgs-Higgs boson couplings. In Sect. 4 we apply the general formalism to models with fermion content as in the SM and with one $SU(2)$ Higgs singlet and any number l of Higgs doublets. In Sect. 5 we show how such models with $l \geq 3$ Higgs doublets provide all the prerequisites for generating chirality-conserving, CP-violating effective $ZbbG$ couplings at the one-loop level. Section 6 contains our conclusions. In the appendices we discuss some properties of the general $SU(2) \times U(1)$ representation carried by scalar fields.

2 The general Higgs field and spontaneous symmetry breaking

We consider a gauge theory based on the electroweak gauge group $G = SU(2) \times U(1)$ (for our notation cf. [8]). The elements of G will be denoted by U . A suitable concrete realization of G is by 2×2 matrices with the following parametrization:

$$U(\varphi, \psi) = \exp \left[i \frac{1}{2} \tau_a \varphi_a + iy_0 \psi \right] \quad (2.1)$$

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¹ Other scenarios for electroweak symmetry breaking like technicolor models have been discussed [6], but remain less well-developed theoretically and seem to be disfavoured by the data; see, e.g., [7]

where τ_a ($a = 1, 2, 3$) are the Pauli matrices and $\boldsymbol{\varphi} = (\varphi_a)$ where is restricted to

$$|\boldsymbol{\varphi}| < 2\pi. \quad (2.2)$$

We assume y_0^{-1} to be a natural number ($y_0^{-1} = 1, 2, \dots$) which will be chosen conveniently later on, and we have

$$|\boldsymbol{\psi}| < \pi y_0^{-1}. \quad (2.3)$$

The parametrization (2.1) is almost everywhere regular on G . For our purposes below it suffices to note that (2.1) is regular in a suitable neighbourhood of the unit element of G : $U = \mathbb{1}_2$. Taking an arbitrary element $U_0 \in G$, we get a parametrization of the elements of G which is regular in a neighbourhood of U_0 by setting

$$U = U(\boldsymbol{\varphi}, \boldsymbol{\psi}) \cdot U_0. \quad (2.4)$$

From (2.1) to (2.4) we see that G is a differentiable, compact manifold.

In the following χ denotes a Higgs field that transforms under G according to an arbitrary representation, which is in general reducible and contains real orthogonal as well as complex unitary parts². Let us first show that without loss of generality we can assume χ to carry a real orthogonal representation of G . To see this, consider a Higgs field³ ϕ carrying a unitary representation of dimension r :

$$\phi = \begin{pmatrix} \phi_1 \\ \cdot \\ \cdot \\ \cdot \\ \phi_r \end{pmatrix}, \quad \phi \in \mathbb{C}_r \quad (2.5)$$

where the action of G is as follows:

$$U : \phi \longrightarrow D_r(U)\phi, \quad (2.6)$$

$$\begin{aligned} D_r^\dagger(U)D_r(U) &= \mathbb{1}_r, \\ U &\in G. \end{aligned} \quad (2.7)$$

We define a corresponding $2r$ component real Higgs field χ by setting

$$\begin{aligned} \chi &= (\chi_{\alpha,j}) \\ (\alpha &= 1, 2; \quad j = 1, \dots, r), \\ \chi_{1,j} &:= \text{Re}\phi_j, \\ \chi_{2,j} &:= \text{Im}\phi_j. \end{aligned} \quad (2.8)$$

Furthermore we define the real $2r \times 2r$ matrices

$$R_{2r}(U) := \mathbb{1}_2 \otimes \text{Re}D_r(U) - \epsilon \otimes \text{Im} D_r(U) \quad (2.9)$$

² We always stay in the framework of local relativistic quantum field theory. Thus for instance self-conjugate scalar fields of half integral $SU(2)$ representations which exist in nonlocal theories [9] are not considered

³ Here and in the following we suppress the space-time variable x , if there is no danger of misunderstanding. Thus $\phi \in \mathbb{C}_r$ in (2.5) and below is to be read as $\phi(x) \in \mathbb{C}_r$ for each x . Likewise we introduce in (2.8) a $2r$ component real vector $\chi(x)$

where

$$\epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \quad (2.10)$$

It is easy to see that we have

$$\phi^\dagger \phi = \chi^T \chi. \quad (2.11)$$

The transformation (2.6) corresponds to

$$U : \chi \longrightarrow R_{2r}(U)\chi \quad (2.12)$$

and

$$U \longrightarrow R_{2r}(U)$$

is a real orthogonal representation of G :

$$\begin{aligned} R_{2r}(U)R_{2r}(U') &= R_{2r}(UU'), \\ R_{2r}(U^\dagger) &= R_{2r}^T(U), \\ R_{2r}^T(U)R_{2r}(U) &= \mathbb{1}_{2r}, \\ (U, U' \in G). \end{aligned} \quad (2.13)$$

We can thus start with a general real Higgs field χ with n components

$$\chi = \begin{pmatrix} \chi_1 \\ \cdot \\ \cdot \\ \cdot \\ \chi_n \end{pmatrix}, \quad (2.14)$$

carrying an orthogonal representation of G :

$$\begin{aligned} U : \chi &\longrightarrow R(U)\chi, \\ R^T(U)R(U) &= \mathbb{1}. \end{aligned} \quad (2.15)$$

For infinitesimal transformations $U(\delta\boldsymbol{\varphi}, \delta\boldsymbol{\psi})$ (cf. (2.1)) we have

$$R(U(\delta\boldsymbol{\varphi}, \delta\boldsymbol{\psi})) = \mathbb{1} + \delta\varphi_a \tilde{T}_a + \delta\psi \tilde{Y}. \quad (2.16)$$

Here \tilde{T}_a and \tilde{Y} are the real antisymmetric matrices representing the generators of G :

$$\begin{aligned} \tilde{T}_a + \tilde{T}_a^T &= 0, \\ \tilde{Y} + \tilde{Y}^T &= 0; \end{aligned} \quad (2.17)$$

$$\begin{aligned} [\tilde{T}_a, \tilde{T}_b] &= -\epsilon_{abc} \tilde{T}_c, \\ [\tilde{Y}, \tilde{T}_a] &= 0. \end{aligned} \quad (2.18)$$

We define the corresponding generator of the electromagnetic gauge group $U(1)_{em}$ as usual by

$$\tilde{Q} = \tilde{T}_3 + \tilde{Y}. \quad (2.19)$$

We note the commutation relations following from (2.18) and (2.19):

$$\begin{aligned} [\tilde{Q}, \tilde{T}_1] &= -\tilde{T}_2, \\ [\tilde{Q}, \tilde{T}_2] &= \tilde{T}_1, \\ [\tilde{Q}, \tilde{T}_3] &= 0. \end{aligned} \quad (2.20)$$

Consider next the matrix $\tilde{Y}^T \tilde{Y}$ which is symmetric and positive semi-definite. We assume that the eigenvalues y_j^2 of $\tilde{Y}^T \tilde{Y}$ satisfy

$$|y_j| \cdot y_0^{-1} = \text{integer} \quad (j = 1, \dots, n). \quad (2.21)$$

The representation of G in the space of Higgs fields is then single-valued (cf. Appendix A).

The reason for going through these subtleties here is because we want to avoid the case where the Higgs representation of G is only single-valued when considered as a representation of the universal covering group of G , which is not a compact manifold. This would happen if the ratio of two numbers $|y_j|$ and $|y_k|$ was irrational.

We shall now study the Higgs part of the Lagrangian describing an $SU(2) \times U(1)$ gauge theory with the arbitrary Higgs field χ :

$$\mathcal{L}_\chi = \frac{1}{2} (D_\mu \chi)^T (D^\mu \chi) - V(\chi). \quad (2.22)$$

Here

$$D_\mu \chi := (\partial_\mu + g W_\mu^a \tilde{T}_a + g' B_\mu \tilde{Y}) \chi \quad (2.23)$$

is the covariant derivative of χ and g and g' are the $SU(2)$ and $U(1)$ coupling constants, respectively. The gauge boson fields are denoted by W_μ^a and B_μ . The Higgs potential $V(\chi)$ must be invariant under G and is constrained by the requirements of hermiticity and renormalizability. Thus V can contain up to fourth powers of χ .

In the following we let V largely unspecified apart from assuming that it leads to spontaneous symmetry breaking where only the electromagnetic gauge group $U(1)_{em}$ remains unbroken. Let v be the vector of vacuum expectation values of χ (at tree level):

$$v = \langle 0 | \chi | 0 \rangle \neq 0. \quad (2.24)$$

We must then have

$$\tilde{Q} v = 0, \quad (2.25)$$

and the three vectors

$$\tilde{T}_a v \quad (a = 1, 2, 3) \quad (2.26)$$

must be linearly independent. Using (2.17)-(2.20) it is easy to derive the following relations:

$$v^T \tilde{T}_a v = 0, \quad (a = 1, 2, 3), \quad (2.27)$$

$$\begin{aligned} v^T \tilde{T}_1 \tilde{T}_1 v &= v^T \tilde{T}_2 \tilde{T}_2 v, \\ v^T \tilde{T}_1 \tilde{T}_2 v &= -v^T \tilde{T}_2 \tilde{T}_1 v, \end{aligned} \quad (2.28)$$

$$\begin{aligned} v^T \tilde{T}_3 \tilde{T}_3 v &= v^T \tilde{Y} \tilde{Y} v, \\ v^T \tilde{T}_3 \tilde{Y} v &= -v^T \tilde{Y} \tilde{T}_3 v. \end{aligned} \quad (2.29)$$

Let us next define the shifted Higgs field χ' by

$$\chi' := \chi - v. \quad (2.30)$$

We get then

$$\begin{aligned} D_\mu \chi &= (\partial_\mu + \tilde{\Omega}_\mu) \chi \\ &= \tilde{\Omega}_\mu v + \partial_\mu \chi' + \tilde{\Omega}_\mu \chi', \end{aligned} \quad (2.31)$$

where

$$\tilde{\Omega}_\mu := g W_\mu^a \tilde{T}_a + g' B_\mu \tilde{Y}. \quad (2.32)$$

In terms of the physical vector boson fields Z_μ, A_μ we have ($s \equiv \sin \theta_w$, $c \equiv \cos \theta_w$, $e = gs = g'c$):

$$\begin{aligned} Z_\mu &= c W_\mu^3 - s B_\mu, \\ A_\mu &= s W_\mu^3 + c B_\mu; \end{aligned} \quad (2.33)$$

$$\tilde{\Omega}_\mu = \frac{e}{s} \left(W_\mu^1 \tilde{T}_1 + W_\mu^2 \tilde{T}_2 \right) + \frac{e}{sc} Z_\mu \left(\tilde{T}_3 - s^2 \tilde{Q} \right) + e A_\mu \tilde{Q}. \quad (2.34)$$

With this the Lagrangian \mathcal{L}_χ (2.22) reads:

$$\begin{aligned} \mathcal{L}_\chi &= \frac{1}{2} v^T \tilde{\Omega}_\mu^T \tilde{\Omega}^\mu v + \frac{1}{2} \partial_\mu \chi'^T \partial^\mu \chi' - v^T \tilde{\Omega}_\mu \partial^\mu \chi' \\ &\quad - \chi'^T \tilde{\Omega}_\mu \partial^\mu \chi' - v^T \tilde{\Omega}_\mu \tilde{\Omega}^\mu \chi' + \frac{1}{2} \chi'^T \tilde{\Omega}_\mu^T \tilde{\Omega}^\mu \chi' \\ &\quad - V(v + \chi'). \end{aligned} \quad (2.35)$$

The successive terms on the r.h.s. of (2.35) will be denoted by $\mathcal{L}_\chi^{(i)}$, $i = 1, \dots, 7$.

Aspects of such a Lagrangian (2.35) have been studied previously, for instance the $Z - W$ -Higgs coupling in [10, 11], Higgs triplets in [12, 13], and radiative corrections for models with Higgs triplets in [14, 15].

Let us first study the term bilinear in the vector boson and Higgs fields in (2.35).

$$\begin{aligned} \mathcal{L}_\chi^{(3)} &:= -v^T \tilde{\Omega}_\mu \partial^\mu \chi' \\ &= -\frac{e}{s} \left[W_\mu^1 \partial^\mu (v^T \tilde{T}_1 \chi') + W_\mu^2 \partial^\mu (v^T \tilde{T}_2 \chi') \right] \\ &\quad - \frac{e}{sc} Z_\mu \partial^\mu (v^T \tilde{T}_3 \chi'). \end{aligned} \quad (2.36)$$

In order to discuss the particle content and the couplings of physical particles, it is convenient to use the unitary gauge (for a review and an extensive use of this gauge cf. [5]), which is defined by the condition:

$$v^T \tilde{T}_a \chi' = 0 \quad \text{for } a = 1, 2, 3. \quad (2.37)$$

Can (2.37) always be met? An affirmative answer to this question was given in [5] for the case of a compact group. The proof of [5] goes through also in our case since G is also compact and, by the condition (2.21), we have excluded multivalued representations of $SU(2) \times U(1)$ which would force us to go to the non-compact universal covering group.

To recall the construction of [5] we note first that due to (2.27) the condition (2.37) is equivalent to

$$v^T \tilde{T}_a \chi = 0 \quad \text{for } a = 1, 2, 3. \quad (2.38)$$

Since $v^T \tilde{T}_a$ ($a = 1, 2, 3$) are linearly independent (cf. (2.26)), (2.38) defines a $n - 3$ dimensional linear subspace $\mathbb{R}_{n-3} \subset \mathbb{R}_n$.

$$\mathbb{R}_{n-3} = \{ \chi \mid v^T \tilde{T}_a \chi = 0 \quad \text{for } a = 1, 2, 3 \}. \quad (2.39)$$

Let now χ be an arbitrary vector in \mathbb{R}_n and consider the following real function on the compact manifold G

$$U \rightarrow f(U) = v^T R(U) \chi \quad (U \in G). \quad (2.40)$$

$U_0 \in G$ which maximises $f(U)$ transforms the arbitrary vector $\chi \in \mathbb{R}_n$ into a vector $R(U_0)\chi$ lying in the subspace \mathbb{R}_{n-3} (2.39) of the vectors compatible with the gauge condition. Thus the bilinear couplings of $W_\mu^{1,2}(x)$, $Z_\mu(x)$ to the Higgs field $\chi(x)$ in (2.36) can at any space time point x be rotated away by a suitable transformation $U_0(x) \in G$, where $U_0(x)$ will in general, of course, depend on x . In Appendix B we discuss further some properties of the gauge orbits of our scalar fields.

Having disposed of the bilinear vector boson-Higgs field part $\mathcal{L}_\chi^{(3)}$ of \mathcal{L}_χ (2.35) with the help of the gauge condition (2.37) we turn next to the term $\mathcal{L}_\chi^{(1)}$, bilinear in the vector boson fields, i.e., the vector boson mass term. From (2.35), (2.34) and using (2.28), (2.29), we find

$$\begin{aligned} \mathcal{L}_\chi^{(1)} &= \frac{1}{2} v^T \tilde{\Omega}_\mu^T \tilde{\Omega}^\mu v \\ &= \frac{1}{2} \left(\frac{e}{s}\right)^2 \frac{1}{2} v^T (\tilde{T}_1^T \tilde{T}_1 + \tilde{T}_2^T \tilde{T}_2) v \\ &\quad \cdot (W_\mu^1 W^{1\mu} + W_\mu^2 W^{2\mu}) \\ &\quad + \frac{1}{2} \left(\frac{e}{sc}\right)^2 v^T \tilde{T}_3^T \tilde{T}_3 v \cdot Z_\mu Z^\mu. \end{aligned} \quad (2.41)$$

From (2.41) we can read off the W and Z masses (at tree level)

$$\begin{aligned} m_W^2 &= \left(\frac{e}{s}\right)^2 \frac{1}{2} v^T (\tilde{T}_1^T \tilde{T}_1 + \tilde{T}_2^T \tilde{T}_2) v \\ &= \left(\frac{e}{s}\right)^2 \frac{1}{2} v^T (\tilde{T}_a^T \tilde{T}_a - \tilde{Y}^T \tilde{Y}) v, \\ m_Z^2 &= \left(\frac{e}{sc}\right)^2 v^T \tilde{T}_3^T \tilde{T}_3 v \\ &= \left(\frac{e}{sc}\right)^2 v^T \tilde{Y}^T \tilde{Y} v. \end{aligned} \quad (2.42)$$

This result is of course well known.

Using the decomposition of the representation $U \rightarrow R(U)$ defined in (2.15) but considered as a unitary representation in \mathbb{C}_n as explained in appendix A, we can write (2.42) as follows (cf. (A.7)-(A.16)):

$$\begin{aligned} m_W^2 &= \left(\frac{e}{s}\right)^2 \frac{1}{2} \sum_{t,y} [t(t+1) - y^2] v^T \mathbb{P}(t,y) v, \\ m_Z^2 &= \left(\frac{e}{sc}\right)^2 \sum_{t,y} y^2 v^T \mathbb{P}(t,y) v, \end{aligned} \quad (2.43)$$

where $\mathbb{P}(t,y)$ is the projector on the subspace with representation (t,y) of G . Here t and y are the isospin and hypercharge quantum numbers, respectively. As we see from (A.16), only representations with $y = -t, -t+1, \dots, t$ can contribute with nonzero weight $v^T \mathbb{P}(t,y) v \neq 0$ in the sums (2.43). For the convenience of the reader we have

Table 1. Values for the weak isospin t , the weak hypercharge y and the ρ parameter (2.44), (2.45) for the representations of $G = SU(2) \times U(1)$ with $t \leq 3$ which can give a nonzero contribution in the sums (2.43)

t	y	$t(t+1) - y^2$	y^2	$[t(t+1) - y^2]/(2y^2)$
0	0	0	0	
1/2	$\pm 1/2$	1/2	1/4	1
1	± 1	1	1	1/2
1	0	2	0	∞
3/2	$\pm 3/2$	3/2	9/4	1/3
3/2	$\pm 1/2$	7/2	1/4	7
2	± 2	2	4	1/4
2	± 1	5	1	5/2
2	0	6	0	∞
5/2	$\pm 5/2$	5/2	25/4	1/5
5/2	$\pm 3/2$	13/2	9/4	13/9
5/2	$\pm 1/2$	17/2	1/4	17
3	± 2	8	4	1
3	± 1	11	1	11/2
3	0	12	0	∞

listed in Table 1 the values of (t,y) for $t \leq 3$ satisfying the above condition and the corresponding values for the ρ parameter, defined as usual:

$$\rho := \frac{m_W^2}{m_Z^2 \cos^2 \theta_W}. \quad (2.44)$$

From (2.43) we get

$$\rho = \frac{\sum_{t,y} [t(t+1) - y^2] v^T \mathbb{P}(t,y) v}{2 \sum_{t,y} y^2 v^T \mathbb{P}(t,y) v}. \quad (2.45)$$

Due to the non-negativity of the weights $v^T \mathbb{P}(t,y) v \geq 0$ (cf. (A.17)) the value of ρ for an arbitrary representation of G must be inside the interval spanned by the values of ρ from the irreducible representations contributing with nonzero weight in (2.43). Note that the tree-level relation $\rho = 1$ holds, apart from the Higgs doublet representations $t = 1/2, y = \pm 1/2$, also for the triplet representations $t = 3, y = \pm 2$. (Actually, there are other, higher-dimensional representations satisfying this tree level relation; see for instance [11].)

3 The general structure of the Z -Higgs-Higgs and W -Higgs-Higgs vertices

In this section we derive some properties of the vertices describing the coupling of the Z and W bosons to two physical Higgs particles. In particular we give the conditions for having a non-diagonal real or complex Z -Higgs-Higgs boson coupling.

The corresponding term of the Lagrangian (2.35) is

$$\begin{aligned}\mathcal{L}_\chi^{(4)} &= -\chi'^T \tilde{\Omega}_\mu \partial^\mu \chi' \\ &= -\frac{e}{s} W_\mu^1 \chi'^T \tilde{T}_1 \partial^\mu \chi' - \frac{e}{s} W_\mu^2 \chi'^T \tilde{T}_2 \partial^\mu \chi' \\ &\quad - \frac{e}{sc} Z_\mu \chi'^T (\tilde{T}_3 - s^2 \tilde{Q}) \partial^\mu \chi' \\ &\quad - e A_\mu \chi'^T \tilde{Q} \partial^\mu \chi',\end{aligned}\quad (3.1)$$

where χ' is the shifted Higgs field (cf. (2.30)), a vector in the space \mathbb{R}_{n-3} (2.39). It is convenient to introduce the projector onto this space of the physical Higgs fields. For this we define 3 vectors w_a ($a = 1, 2, 3$) in \mathbb{R}_n :

$$w_a := \tilde{T}_a v \cdot (v^T \tilde{T}_a^T \tilde{T}_a v)^{-1/2} \quad (\text{no summation over } a). \quad (3.2)$$

From (2.28), (2.29) and (2.42) we find

$$\begin{aligned}w_1 &= \tilde{T}_1 v \cdot \frac{e}{sm_W}, \\ w_2 &= \tilde{T}_2 v \cdot \frac{e}{sm_W}, \\ w_3 &= \tilde{T}_3 v \cdot \frac{e}{scm_Z},\end{aligned}\quad (3.3)$$

$$w_a^T w_b = \delta_{ab}. \quad (3.4)$$

From (2.39) we see that the vectors w_a are the normalised vectors orthogonal to \mathbb{R}_{n-3} . The projector onto \mathbb{R}_{n-3} is thus given by

$$\mathbb{P}' := \mathbb{1} - w_a w_a^T. \quad (3.5)$$

From (2.20) and (2.25) we get

$$\begin{aligned}\tilde{Q} w_1 &= -w_2, \\ \tilde{Q} w_2 &= w_1, \\ \tilde{Q} w_3 &= 0,\end{aligned}\quad (3.6)$$

which leads to

$$[\mathbb{P}', \tilde{Q}] = 0. \quad (3.7)$$

Thus \mathbb{P}' commutes with the generator of electric charge. In general, however, \mathbb{P}' will not commute with \tilde{T}_3 .

When discussing the couplings in (3.1) we have to restrict the coupling matrices \tilde{T}_a, \tilde{Q} to the space of physical Higgs fields. This can be done with the help of the projector \mathbb{P}' (3.5). We define the following matrices

$$\begin{aligned}\tilde{T}_a'' &:= \mathbb{P}' \tilde{T}_a \mathbb{P}' \quad (a = 1, 2, 3), \\ \tilde{Q}'' &:= \mathbb{P}' \tilde{Q} \mathbb{P}'.\end{aligned}\quad (3.8)$$

The matrices (3.8) are block-diagonal, with non-trivial $(n-3) \times (n-3)$ submatrices $\tilde{T}_a'', \tilde{Q}''$ on \mathbb{R}_{n-3} and zero on its orthogonal complement. In the following we shall only deal with the submatrices on \mathbb{R}_{n-3} . Equations (2.20) and (3.7) imply that

$$[\tilde{T}_3'', \tilde{Q}''] = 0. \quad (3.9)$$

Similarly we find

$$\begin{aligned}[\tilde{Q}', \tilde{T}_1'] &= -\tilde{T}_2', \\ [\tilde{Q}', \tilde{T}_2'] &= \tilde{T}_1'.\end{aligned}\quad (3.10)$$

Note, however, that the matrices \tilde{T}_a' ($a = 1, 2, 3$) will in general not satisfy the $SU(2)$ commutation relations. Our aim is now to diagonalise the matrices \tilde{Q}' and \tilde{T}_3' and to arrange the components of the physical Higgs field χ' into charge eigenstates. However, since \tilde{Q}' and \tilde{T}_3' are antisymmetric real matrices on the real space \mathbb{R}_{n-3} , this requires some nontrivial work similar to the one of Appendix A.

Let us embed the space \mathbb{R}_{n-3} into the complex space \mathbb{C}_{n-3} and define matrices:

$$\begin{aligned}T_a' &= \frac{1}{i} \tilde{T}_a', \quad (a = 1, 2, 3), \\ Q' &= \frac{1}{i} \tilde{Q}'.\end{aligned}\quad (3.11)$$

We have

$$\begin{aligned}\tilde{T}_a'^T &= -\tilde{T}_a', \\ \tilde{Q}'^T &= -\tilde{Q}';\end{aligned}\quad (3.12)$$

$$\begin{aligned}T_a' &= -T_a'^T = T_a'^\dagger = -T_a'^*, \\ Q' &= -Q'^T = Q'^\dagger = -Q'^*;\end{aligned}\quad (3.13)$$

$$[T_3', Q'] = 0. \quad (3.14)$$

It follows from (3.13), (3.14) that the hermitian matrices Q', T_3' can be diagonalized simultaneously in \mathbb{C}_{n-3} . Let the eigenvalue pairs be (q, t_3') . We consider the double resolvent

$$\frac{1}{(\xi - Q')(\eta - T_3')} = \sum_{q, t_3'} \frac{\mathbb{P}(q, t_3')}{(\xi - q)(\eta - t_3')} \quad (\xi, \eta \in \mathbb{C}), \quad (3.15)$$

where $\mathbb{P}(q, t_3')$ is the projector onto the subspace of eigenvectors associated with the eigenvalue pair (q, t_3') . By taking the transposed of (3.15) and using (3.13) we find

$$\frac{1}{(\xi + Q')(\eta + T_3')} = \sum_{q, t_3'} \frac{\mathbb{P}^T(q, t_3')}{(\xi - q)(\eta - t_3')}, \quad (3.16)$$

$$\sum_{q, t_3'} \frac{\mathbb{P}(q, t_3')}{(\xi + q)(\eta + t_3')} = \sum_{q, t_3'} \frac{\mathbb{P}^T(q, t_3')}{(\xi - q)(\eta - t_3')}. \quad (3.17)$$

Comparing the poles and residues on the r.h.s. and l.h.s. of (3.17), we conclude that with (q, t_3') also $(-q, -t_3')$ must be an eigenvalue pair and (q, t_3') and $(-q, -t_3')$ have the same multiplicity. For the projectors we find

$$\mathbb{P}(-q, -t_3') = \mathbb{P}^T(q, t_3'). \quad (3.18)$$

We treat now the eigenspaces with $q = 0$ and with $q \neq 0$ separately. Note that the eigenspace with $q = 0$ must always have dimension ≥ 1 , since

$$\chi' = cv \quad (c \in \mathbb{C}, c \neq 0) \quad (3.19)$$

is an eigenvector of Q' in \mathbb{C}_{n-3} with eigenvalue 0:

$$Q'(cv) = 0. \tag{3.20}$$

Let us denote by S_q the subspace of \mathbb{C}_{n-3} corresponding to charge eigenvalue q and arbitrary t'_3 . The projector onto S_q is

$$\mathbb{P}(q) = \sum_{t'_3} \mathbb{P}(q, t'_3). \tag{3.21}$$

The hermiticity of \mathbb{P} and (3.18) imply that $\mathbb{P}(0)$ is a real, symmetric matrix which can also be considered as a projector in the real space \mathbb{R}_{n-3} . Hence real eigenvectors $u_1, \dots, u_{r_0} \in \mathbb{R}_{n-3}$ exist, such that

$$\begin{aligned} \mathbb{P}(0)u_j &= u_j, \\ Q'u_j &= 0, \\ (j &= 1, \dots, r_0), \\ \mathbb{P}(0) &= \sum_{j=1}^{r_0} u_j u_j^T. \end{aligned} \tag{3.22}$$

A particular set of such eigenvectors u_j can be constructed as follows. In the subspace corresponding to $q = t'_3 = 0$ (if this occurs at all) we take an arbitrary set of normalised real eigenvectors u_j . For $q = 0, t'_3 \neq 0$ we consider the common eigenvectors of Q' and T'_3 in \mathbb{C}_{n-3}

$$\begin{aligned} Q'u(0, t'_3) &= 0, \\ T'_3 u(0, t'_3) &= t'_3 u(0, t'_3). \end{aligned} \tag{3.23}$$

From (3.13) we get

$$\begin{aligned} Q'u^*(0, t'_3) &= 0, \\ T'_3 u^*(0, t'_3) &= -t'_3 u^*(0, t'_3). \end{aligned} \tag{3.24}$$

This shows that $u^*(0, t'_3)$ are eigenvectors to Q', T'_3 with eigenvalues $(0, -t'_3)$. Therefore the set of vectors

$$u(0, t'_3), \quad u^*(0, t'_3), \quad (t'_3 > 0),$$

where we can choose the normalisation such that

$$u^\dagger(0, t'_3)u(0, t''_3) = \delta_{t'_3, t''_3}, \tag{3.25}$$

forms a basis of eigenvectors for $q = 0, t'_3 \neq 0$. From (3.24) we see that we also have

$$u^{*\dagger}(0, t'_3)u(0, t''_3) = 0. \tag{3.26}$$

The vectors

$$\begin{aligned} u_1(0, t'_3) &:= \sqrt{2} \operatorname{Re} u(0, t'_3), \\ u_2(0, t'_3) &:= \sqrt{2} \operatorname{Im} u(0, t'_3), \\ (t'_3 > 0), \end{aligned} \tag{3.27}$$

are then linearly independent, normalised vectors in \mathbb{R}_{n-3} which we can choose as basis vectors satisfying (3.22). In this basis the real, antisymmetric matrix \tilde{T}'_3 has the

following structure in the $q = 0$ subspace: It is block-diagonal, with possibly a number of zeros and then 2×2 matrices

$$\begin{pmatrix} 0 & t'_3 \\ -t'_3 & 0 \end{pmatrix}$$

where t'_3 are the positive eigenvalues of the hermitian matrix T'_3 :

$$\tilde{T}'_3{}^{(0)} = \begin{pmatrix} 0 & & & \\ & \cdot & & \\ & & 0 & 0 \\ & & 0 & \\ \hline & & 0 & t'_3 \\ & & & -t'_3 & 0 \\ \hline & & 0 & & \cdot \\ & & & & \cdot \end{pmatrix}. \tag{3.28}$$

We can consider (3.28) as a standard form for $\tilde{T}'_3{}^{(0)}$, the submatrix of \tilde{T}'_3 in the $q = 0$ subspace.

For $q \neq 0$ we consider simultaneously the subspaces S_q and S_{-q} . We can then without loss of generality assume $q > 0$. Let $u(q, t'_3)$ be the common eigenvectors of Q' and T'_3 (cf.(3.11)) in \mathbb{C}_{n-3}

$$\begin{aligned} Q'u(q, t'_3) &= qu(q, t'_3), \\ T'_3 u(q, t'_3) &= t'_3 u(q, t'_3), \end{aligned} \tag{3.29}$$

where t'_3 runs over all eigenvalues of T'_3 corresponding to charge q and we have suppressed a possible degeneracy index. From (3.13) we get

$$\begin{aligned} Q'u^*(q, t'_3) &= -qu^*(q, t'_3), \\ T'_3 u^*(q, t'_3) &= -t'_3 u^*(q, t'_3). \end{aligned} \tag{3.30}$$

With suitable numbering and phases, the vectors $u^*(q, t'_3)$ can thus be considered as the eigenvectors of Q', T'_3 in S_{-q} associated with the eigenvalue pair $(-q, -t'_3)$. We can normalise $u(q, t'_3)$ to

$$u^\dagger(q, t'_3)u(q, t''_3) = \delta_{t'_3, t''_3} \tag{3.31}$$

and (3.30) implies

$$u^{*\dagger}(q, t'_3)u(q, t''_3) = 0. \tag{3.32}$$

We can define real vectors $u_{1,2}(q, t'_3)$ by

$$\begin{aligned} u_1(q, t'_3) &:= \sqrt{2} \cdot \operatorname{Re} u(q, t'_3), \\ u_2(q, t'_3) &:= \sqrt{2} \cdot \operatorname{Im} u(q, t'_3). \end{aligned} \tag{3.33}$$

From (3.31), (3.32) we find that $u_{1,2}(q, t'_3)$ are linearly independent real vectors in \mathbb{R}_{n-3} satisfying

$$\begin{aligned} u_1^T(q, t'_3)u_1(q, t''_3) &= \delta_{t'_3, t''_3}, \\ u_2^T(q, t'_3)u_2(q, t''_3) &= \delta_{t'_3, t''_3}, \\ u_1^T(q, t'_3)u_2(q, t''_3) &= 0. \end{aligned} \tag{3.34}$$

Let us now choose the basis vectors in \mathbb{R}_{n-3} as follows. In the subspace S_0 we take the u_j of (3.22). In the subspaces $S_q + S_{-q}$ ($q > 0$) we choose the vectors $u_{1,2}$ of (3.33) and denote them collectively by

$$u_{1,\alpha}, u_{2,\alpha}, \quad (\alpha = 1, 2, \dots, r),$$

where α stands for the pair (q, t'_3) plus a possible degeneracy index. We can then decompose any given vector $\chi' \in \mathbb{R}_{n-3}$ as

$$\chi' = \sum_{j=1}^{r_0} \chi'_j u_j + \sum_{\alpha=1}^r (\chi'_{1,\alpha} u_{1,\alpha} + \chi'_{2,\alpha} u_{2,\alpha}) \quad (3.35)$$

and we get

$$\chi'^T \chi' = \sum_{j=1}^{r_0} (\chi'_j)^2 + \sum_{\alpha=1}^r [(\chi'_{1,\alpha})^2 + (\chi'_{2,\alpha})^2]. \quad (3.36)$$

The fields $\chi_{1,\alpha}, \chi_{2,\alpha}$ ($\alpha = 1, \dots, r$) corresponding to $q \neq 0$ can be rearranged into r complex Higgs fields. For this we define

$$\begin{aligned} u_\alpha &:= \frac{1}{\sqrt{2}}(u_{1,\alpha} + i u_{2,\alpha}), \\ \phi'_\alpha &:= \frac{1}{\sqrt{2}}(\chi'_{1,\alpha} - i \chi'_{2,\alpha}). \end{aligned} \quad (3.37)$$

The u_α are the complex eigenvectors $u(q, t'_3)$ of (3.29) which we write now as

$$\begin{aligned} \tilde{Q}' u_\alpha &= i q_\alpha u_\alpha, \\ \tilde{T}'_3 u_\alpha &= i t'_{3,\alpha} u_\alpha, \\ (q_\alpha > 0). \end{aligned} \quad (3.38)$$

We get then

$$\chi' = \sum_{j=1}^{r_0} \chi'_j u_j + \sum_{\alpha=1}^r (\phi'_\alpha u_\alpha + \phi'^*_\alpha u_\alpha^*) \quad (3.39)$$

$$\frac{1}{2} \chi'^T \chi' = \frac{1}{2} \sum_{j=1}^{r_0} \chi'_j \chi'_j + \sum_{\alpha=1}^r \phi'^*_\alpha \phi'_\alpha, \quad (3.40)$$

$$\frac{1}{2} \partial_\mu \chi'^T \partial^\mu \chi' = \frac{1}{2} \sum_{j=1}^{r_0} \partial_\mu \chi'_j \partial^\mu \chi'_j + \sum_{\alpha=1}^r \partial_\mu \phi'^*_\alpha \partial^\mu \phi'_\alpha, \quad (3.41)$$

$$\begin{aligned} \mathcal{L}_\chi^{(4)} &= -e \left\{ A_\mu J_{H,em}^\mu + \frac{1}{s c} Z_\mu J_{H,NC}^\mu \right. \\ &\quad \left. + \frac{1}{\sqrt{2} s} (W_\mu^+ J_{H,CC}^\mu + W_\mu^- J_{HCC}^{\mu\dagger}) \right\} \end{aligned} \quad (3.42)$$

where

$$W_\mu^\pm = \frac{1}{\sqrt{2}} (W_\mu^1 \mp i W_\mu^2) \quad (3.43)$$

and $J_{H,em}^\mu, J_{H,NC}^\mu, J_{H,CC}^\mu$ are the electromagnetic, neutral and charged Higgs currents, respectively:

$$J_{H,em}^\mu = i \sum_{\alpha} q_\alpha \phi'_\alpha{}^* \overleftrightarrow{\partial}^\mu \phi'_\alpha, \quad (3.44)$$

$$J_{H,NC}^\mu = \frac{1}{2} \sum_{j,k} \chi'_j \tilde{T}'_{3,jk}{}^{(0)} \overleftrightarrow{\partial}^\mu \chi'_k + i \sum_{\alpha} (t'_{3,\alpha} - s^2 q_\alpha) \phi'_\alpha{}^* \overleftrightarrow{\partial}^\mu \phi'_\alpha, \quad (3.45)$$

$$\begin{aligned} J_{H,CC}^\mu &= \frac{1}{2} \chi'^T (\tilde{T}'_1 + i \tilde{T}'_2) \overleftrightarrow{\partial}^\mu \chi', \\ &(\overleftrightarrow{\partial}^\mu = \overrightarrow{\partial}^\mu - \overleftarrow{\partial}^\mu) \end{aligned} \quad (3.46)$$

The fields χ'_j ($j = 1, \dots, r_0$) correspond to neutral particles, and the fields ϕ'_α annihilate (create) particles of charge $e q_\alpha$ ($-e q_\alpha$). In (3.44)-(3.46) we have thus used a basis where the electromagnetic current and the contribution of the charged fields to the neutral current are diagonal. The part of the neutral current from neutral fields is written in the basis where $\tilde{T}'_3{}^{(0)}$ is in the standard form (3.28).

Of course, the above basis is, in general, not identical to the mass eigenbasis for the Higgs fields. The Higgs boson mass matrix \mathcal{M} is obtained from the bilinear term in the potential V (2.22) expressed in terms of the shifted fields χ' (2.30):

$$V(v + \chi') = V(v) + \frac{1}{2} \chi'^T \mathcal{M} \chi' + \dots \quad (3.47)$$

Let

$$\hat{\chi}_j \quad (j = 1, \dots, r_0)$$

be the real fields diagonalising this mass matrix in the neutral ($q = 0$) sector and let

$$\hat{\phi}_{q,\alpha} \quad (\alpha = 1, \dots, r_q)$$

be the complex mass eigenfields carrying charge $q > 0$. We have then

$$\hat{\chi} \equiv \begin{pmatrix} \hat{\chi}_1 \\ \vdots \\ \hat{\chi}_{r_0} \end{pmatrix} = V^{(0)T} \begin{pmatrix} \chi'_1 \\ \vdots \\ \chi'_{r_0} \end{pmatrix}, \quad (3.48)$$

$$\hat{\Phi}_q \equiv \begin{pmatrix} \hat{\phi}_{q,1} \\ \vdots \\ \hat{\phi}_{q,r_q} \end{pmatrix} = V^{(q)\dagger} \begin{pmatrix} \phi'_{q,1} \\ \vdots \\ \phi'_{q,r_q} \end{pmatrix}, \quad (3.49)$$

where $V^{(0)}$ is a real orthogonal $r_0 \times r_0$ and the $V^{(q)}$ are unitary $r_q \times r_q$ matrices. Of course, only Higgs fields of the same charge mix. Inserting (3.48) and (3.49) in (3.44),

(3.45), we get the following expressions for the electromagnetic and neutral Higgs currents in terms of mass eigenfields:

$$J_{H,em}^\mu = i \sum_{q>0} q \hat{\Phi}_q^\dagger \overleftrightarrow{\partial}^\mu \hat{\Phi}_q, \quad (3.50)$$

$$J_{H,NC}^\mu = \frac{1}{2} \sum_{j,k} \hat{\chi}_j \hat{T}_{3,jk}^{(0)} \overleftrightarrow{\partial}^\mu \hat{\chi}_k + i \sum_{q>0} \hat{\Phi}_q^\dagger (\hat{T}_3^{(q)} - s^2 q) \overleftrightarrow{\partial}^\mu \hat{\Phi}_q. \quad (3.51)$$

Here we have for $q = 0$

$$\hat{T}_3^{(0)} = V^{(0)T} \tilde{T}_3^{(0)} V^{(0)}, \quad (3.52)$$

and for $q > 0$

$$\hat{T}_3^{(q)} = V^{(q)\dagger} \cdot \text{diag} (t'_{3,1}{}^{(q)}, \dots, t'_{3,r_q}{}^{(q)}) \cdot V^{(q)} \quad (3.53)$$

with $t'_{3,1}{}^{(q)} \dots, t'_{3,r_q}{}^{(q)}$ the eigenvalues of T_3' occurring for charge q . The matrices $V^{(q)}$ ($q > 0$) are the analogues of the Cabibbo-Kobayashi-Maskawa (CKM) matrix [16] governing the quark-W-boson couplings. There is still some freedom in the choice of $V^{(q)}$, i.e. we can make the replacement

$$V^{(q)} \longrightarrow U_1^{(q)\dagger} V^{(q)} U_2^{(q)}, \quad (3.54)$$

where $U_1^{(q)}$ is a unitary matrix commuting with $\text{diag} (t'_{3,1}{}^{(q)}, \dots, t'_{3,r_q}{}^{(q)})$ and $U_2^{(q)}$ is a unitary matrix commuting with the mass matrix for charge q . In particular if, and only if, all mass eigenvalues and all $t'_{3,\alpha}{}^{(q)}$ corresponding to charge q are different then $U_{1,2}^{(q)}$ are diagonal unitary matrices (apart from trivial renumbering of the fields). Thus the questions concerning diagonal versus non-diagonal real/complex couplings in the neutral current involving physical Higgs fields can immediately be answered: We find the following for charged physical Higgs fields:

(1) A non-diagonal Z -Higgs-Higgs boson coupling requires at least two Higgs fields with the same charge, but different mass, being linearly related to fields with two different eigenvalues $t'_{3,\alpha}{}^{(q)}$ of the matrix T_3' in the sector corresponding to charge q . Here T_3' is related by a projection (cf. (3.8) ff.) to the matrix of the third component of the weak isospin.

(2) If only two charged fields mix, the mixing matrix $V^{(q)}$ can always be made real by a replacement (3.54). Thus, in this case, the non-diagonal Z -Higgs-Higgs boson coupling can, without loss of generality, be assumed to be real.

(3) A non-diagonal complex Z -Higgs-Higgs coupling (whose phase(s) cannot be rotated away) for charged Higgs bosons requires at least three Higgs fields of the same charge with different masses, where these fields are linearly related to fields with at least three different eigenvalues $t'_{3,\alpha}{}^{(q)}$. For 3 Higgs fields the form of the mixing matrix $V^{(q)}$ can be chosen in analogy to the CKM matrix for 3 quark generations.

In the neutral Higgs sector the $Z\hat{\chi}\hat{\chi}$ coupling matrix $\hat{T}_3^{(0)}$ (3.52) is, in general, an arbitrary antisymmetric matrix. Indeed, $V^{(0)}$ in (3.48) can be an arbitrary orthogonal matrix and any antisymmetric matrix $\hat{T}_3^{(0)}$ can be brought to the standard form (3.28) by an orthogonal transformation. On general grounds we did not find a restriction on the possible values of t'_3 .

The charged scalar current (3.46) can also be straightforwardly expressed in terms of the physical Higgs fields. From (3.10) we have

$$[\tilde{Q}', \tilde{T}'_1 \pm i\tilde{T}'_2] = \pm i(\tilde{T}'_1 \pm i\tilde{T}'_2). \quad (3.55)$$

This guarantees that only fields differing by one unit of charge couple in $J_{H,CC}^\mu$, as it must be by charge conservation. Otherwise we did not find any useful general statement for this current.

4 A model with one Higgs singlet and an arbitrary number of Higgs doublets

In this section we consider as a specific example a model with one complex Higgs singlet ϕ_0 of hypercharge $y = 1$ and l doublets ϕ_j ($j = 1, \dots, l$) of hypercharge $y = 1/2$. The Higgs boson Lagrangian (2.22) is then

$$\mathcal{L}_\chi = (D_\mu \phi_0)^\dagger D^\mu \phi_0 + \sum_{j=1}^l (D_\mu \phi_j)^\dagger (D^\mu \phi_j) - V, \quad (4.1)$$

$$\begin{aligned} D_\mu \phi_0 &= (\partial_\mu + ig' B_\mu) \phi_0, \\ D_\mu \phi_j &= (\partial_\mu + ig W_\mu^a \frac{1}{2} \tau^a + ig' \frac{1}{2} B_\mu) \phi_j, \\ &(j = 1, \dots, l), \end{aligned} \quad (4.2)$$

$$\phi_j = \begin{pmatrix} \phi_{1/2,j} \\ \phi_{-1/2,j} \end{pmatrix}, \quad (4.3)$$

$$V = V_2 + V_3 + V_4, \quad (4.4)$$

$$\begin{aligned} V_2 &= \mu \phi_0^\dagger \phi_0 + \sum_{j,k=1}^l \lambda_{jk} \phi_j^\dagger \phi_k, \\ &(\mu^* = \mu, \quad \lambda_{jk}^* = \lambda_{kj}), \end{aligned} \quad (4.5)$$

$$\begin{aligned} V_3 &= \sum_{j,k=1}^l (\kappa_{jk} \phi_0^\dagger \phi_j^T \epsilon \phi_k + \text{h.c.}), \\ &(\kappa_{jk} = -\kappa_{kj}, \quad \epsilon \text{ as in (2.10)}), \end{aligned} \quad (4.6)$$

$$V_4 = \eta_0 (\phi_0^\dagger \phi_0)^2 + \sum_{j,k=1}^l \eta_{jk} \phi_j^\dagger \phi_k \phi_0^\dagger \phi_0$$

$$\begin{aligned}
 & + \sum_{j,k,r,s=1}^l \left[\xi_{jkr s} (\phi_j^\dagger \epsilon \phi_k^{\dagger T}) (\phi_r^T \epsilon \phi_s) \right. \\
 & \left. + \zeta_{jkr s} (\phi_j^\dagger \tau^a \epsilon \phi_k^{\dagger T}) (\phi_r^T \epsilon \tau^a \phi_s) \right] \\
 & (\eta_0^* = \eta_0; \eta_{jk}^* = \eta_{kj}; \\
 & \xi_{jk,rs} = -\xi_{kj,rs} = -\xi_{jk,rs} = \xi_{sr,kj}^*; \\
 & \zeta_{jk,rs} = \zeta_{kj,rs} = \zeta_{jk,rs} = \zeta_{sr,kj}^*). \quad (4.7)
 \end{aligned}$$

We note that the general form of the Lagrangian (4.1) is unchanged if we make a $U(l)$ transformation on the l Higgs doublets:

$$\phi_j \longrightarrow U_{jk} \phi_k. \quad (4.8)$$

The vacuum expectation values of the Higgs fields are

$$\begin{aligned}
 & \langle 0 | \phi_0 | 0 \rangle = 0, \\
 & \langle 0 | \phi_j | 0 \rangle = \begin{pmatrix} 0 \\ v_j \end{pmatrix}, \\
 & (j = 1, \dots, l), \quad (4.9)
 \end{aligned}$$

where the v_j are complex numbers in general. But by a $U(l)$ transformation (4.8) we can always rotate the Higgs fields such that only v_1 differs from zero and $v_1 > 0$. Then v_1 must have the SM value $\sqrt{2}v_1 = (\sqrt{2}G_F)^{-1/2} = 246$ GeV where G_F is Fermi's constant.

The gauge condition (2.37) reduces then to a condition for the first Higgs doublet ϕ_1 – as in the SM. Thus we get the following set of Higgs fields after spontaneous symmetry breaking: l complex fields of charge 1:

$$\Phi = \begin{pmatrix} \phi_0 \\ \phi_{1/2,2} \\ \cdot \\ \cdot \\ \cdot \\ \phi_{1/2,l} \end{pmatrix}, \quad (4.10)$$

and $2l + 1$ real fields of charge 0:

$$\chi' = \begin{pmatrix} \sqrt{2}\text{Re}(\phi_{-1/2,1} - v_1) \\ \sqrt{2}\text{Re}\phi_{-1/2,2} \\ \sqrt{2}\text{Im}\phi_{-1/2,2} \\ \cdot \\ \cdot \\ \sqrt{2}\text{Re}\phi_{-1/2,l} \\ \sqrt{2}\text{Im}\phi_{-1/2,l} \end{pmatrix}. \quad (4.11)$$

In the following we will discuss mainly the charged fields (4.10) further. As we can easily see from (4.4)-(4.7) their mass matrix $\mathcal{M}^{(1)}$ has the following structure:

$$\mathcal{M}^{(1)} = \begin{pmatrix} \mathcal{M}_{11}^{(1)} & \mathcal{M}_{1k}^{(1)} \\ \mathcal{M}_{j1}^{(1)} & \mathcal{M}_{jk}^{(1)} \end{pmatrix}, \quad (2 \leq j, k \leq l), \quad (4.12)$$

where

$$\mathcal{M}_{1k}^{(1)} = -2v_1 \kappa_{1k}, \quad (k = 2, \dots, l). \quad (4.13)$$

Thus, the V_3 term of the potential in (4.6) induces a mixing of the singlet with the charged components of the doublet Higgs fields.

Let us now consider the neutral current $J_{H,NC}^\mu$ (3.45) in our model. In the basis (4.10), (4.11) it reads

$$J_{H,NC}^\mu = \frac{1}{2} \chi'^T \hat{T}_3^{(0)} \overset{\leftrightarrow}{\partial}^\mu \chi' + i \Phi^\dagger (\hat{T}_3^{(1)} - s^2) \overset{\leftrightarrow}{\partial}^\mu \Phi, \quad (4.14)$$

where

$$\begin{aligned}
 \hat{T}_3^{(0)} &= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & +1/2 & 0 \\ 0 & -1/2 & 0 & 0 \\ 0 & 0 & \cdot & 0 \\ 0 & 0 & \cdot & 0 \\ 0 & 0 & 0 & 0 & +1/2 \\ 0 & 0 & 0 & -1/2 & 0 \end{pmatrix}, \\
 \hat{T}_3^{(1)} &= \begin{pmatrix} 0 & 0 \\ 0 & \frac{1}{2} \delta_{jk} \end{pmatrix}, \quad (2 \leq j, k \leq l). \quad (4.15)
 \end{aligned}$$

Following the discussion in Sect. 3, we transform now to the mass eigenbasis of the Higgs fields according to (3.48), (3.49):

$$\begin{aligned}
 \hat{\chi} &= V^{(0)T} \chi', \\
 \hat{\Phi} &= V^{(1)\dagger} \Phi. \quad (4.16)
 \end{aligned}$$

Since $\hat{T}_3^{(1)}$ in (4.15) has only 2 different eigenvalues, the mixing problem for the charged fields is as simple as in the case of 2 fields. It is easily shown (cf. Appendix C) that $V^{(1)}$ can always be chosen to be a real matrix. Then in terms of the mass eigenfields the neutral current reads

$$\begin{aligned}
 J_{H,NC}^\mu &= \frac{1}{2} \hat{\chi}^T \hat{T}_3^{(0)} \overset{\leftrightarrow}{\partial}^\mu \hat{\chi} + i \hat{\Phi}^\dagger (\hat{T}_3^{(1)} - s^2) \overset{\leftrightarrow}{\partial}^\mu \hat{\Phi}, \\
 \hat{T}_3^{(0)} &= V^{(0)T} \hat{T}_3^{(0)} V^{(0)}, \\
 \hat{T}_3^{(1)} &= V^{(1)\dagger} \hat{T}_3^{(1)} V^{(1)}, \\
 &= \left(\frac{1}{2} \delta_{jk} - \frac{1}{2} V_{1j}^{(1)} V_{1k}^{(1)} \right), \\
 &(1 \leq j, k \leq l). \quad (4.17)
 \end{aligned}$$

In this model we get thus in accordance with our general discussion non-diagonal real Z -Higgs-Higgs boson couplings for the physical Higgs fields. In the next section we will use this model in order to discuss the possibility of producing at one-loop level a CP-violating coupling $Zb\bar{b}G$ which is chirality-conserving and not suppressed by a factor proportional to the b -mass.

5 A chirality-conserving CP-violating $Zb\bar{b}G$ coupling

In [17] the possibility of obtaining an effective CP-violating and chirality-conserving coupling $Zb\bar{b}G$

$$\mathcal{L}_{eff,CP}(x) = \bar{b}(x)T^a\gamma^\nu[h_{Vb} + h_{Ab}\gamma_5]b(x)Z^\mu(x)G_{\mu\nu}^a(x) \quad (5.1)$$

in renormalizable theories at one-loop level was discussed. Here b, Z^μ are the b quark and Z boson fields, $T^a = \lambda^a/2$ are the generators of $SU(3)_c$ and $G_{\mu\nu}^a$ is the gluon field strength tensor. In particular it was shown in [17] that in suitable models, called type I and II, the effective couplings h_{Vb}, h_{Ab} remained nonzero in the limit $m_b \rightarrow 0$. In this section we give an explicit example of a type I model based on the discussion in sect. 4.

Let us start by writing down the most general $SU(2) \times U(1)$ -invariant Yukawa interaction for quarks in the model of Sect. 4:

$$\mathcal{L}_{Yuk} = \sum_{j=1}^l \sum_{\alpha,\beta=1}^3 \left\{ -\bar{d}'_{\alpha R} C_{\alpha\beta}^j \phi_j^\dagger \begin{pmatrix} u_\beta \\ d'_\beta \end{pmatrix}_L + \bar{u}_{\alpha R} C'_{\alpha\beta}{}^j \phi_j^T \epsilon \begin{pmatrix} u_\beta \\ d'_\beta \end{pmatrix}_L + \text{h.c.} \right\} \quad (5.2)$$

Here $\alpha, \beta = 1, 2, 3$ are generation indices, $C_{\alpha\beta}^j$ and $C'_{\alpha\beta}{}^j$ are arbitrary complex numbers, u_α, d'_α denote u -type and d -type fields in the weak isospin basis, and $q_{R,L} = (1 \pm \gamma_5)q/2$. After spontaneous symmetry breaking and transformation to the mass eigenbasis for the quark fields (5.2) reduces to

$$\begin{aligned} \mathcal{L}_{Yuk} = & - \sum_{\alpha=1}^3 [m_{d\alpha} \bar{d}_\alpha d_\alpha + m_{u\alpha} \bar{u}_\alpha u_\alpha] \left(1 + \frac{\chi'_1}{v_1 \sqrt{2}}\right) \\ & + \sum_{j=2}^l \sum_{\alpha,\beta=1}^3 \left\{ - \sum_{\rho=1}^3 \bar{d}_{\rho R} V_{\rho\alpha}^\dagger C_{\alpha\beta}^j \phi_j^\dagger \right. \\ & \times \begin{pmatrix} u_\beta \\ \sum_{\gamma=1}^3 V_{\beta\gamma} d_\gamma \end{pmatrix}_L \\ & \left. + \bar{u}_{\alpha R} C'_{\alpha\beta}{}^j \phi_j^T \epsilon \begin{pmatrix} u_\beta \\ \sum_{\gamma=1}^3 V_{\beta\gamma} d_\gamma \end{pmatrix}_L + \text{h.c.} \right\}. \end{aligned} \quad (5.3)$$

Here $V = (V_{\beta\gamma})$ is the CKM matrix and C, C' denote the $U(l)$ -transformed Yukawa coupling matrices.

The general Yukawa interaction (5.3) leads to flavour-changing neutral currents (FCNC). In order to comply with experimental bounds on FCNC processes one may either impose an appropriate discrete symmetry on \mathcal{L}_{Yuk} or fine-tuning of C, C' is required. Here our aim is to demonstrate a certain property of the Yukawa couplings of charged Higgs bosons to the third quark-generation, namely (5.10) below. For this purpose we discuss, as an example, a model where only the right-handed top quark couples to all the physical Higgs fields. This is realized by

setting

$$\begin{aligned} C_{\alpha\beta}^j &= 0, \\ C'_{\alpha\beta}{}^j &= -\frac{m_t}{v_1} \delta_{\alpha 3} \delta_{\beta 3} \beta_j', \\ & (j = 2, \dots, l). \end{aligned} \quad (5.4)$$

where m_t is the top quark mass and β_j' are arbitrary complex numbers. This leads to

$$\begin{aligned} \mathcal{L}_{Yuk} = & - \sum_{\alpha=1}^3 [m_{d\alpha} \bar{d}_\alpha d_\alpha + m_{u\alpha} \bar{u}_\alpha u_\alpha] \left(1 + \frac{\chi'_1}{v_1 \sqrt{2}}\right) \\ & - \sum_{j=2}^l \left\{ \frac{m_t}{v_1} \beta_j' \bar{t}_R \left[\phi_{1/2,j} \sum_{\alpha=1}^3 V_{3\alpha} d_{\alpha L} - \phi_{-1/2,j} t_L \right] \right. \\ & \left. + \text{h.c.} \right\} \end{aligned} \quad (5.5)$$

In this way we have no flavour-changing neutral interactions at tree level. Transforming to the mass eigenbasis for the Higgs fields according to (4.16) gives:

$$\begin{aligned} \mathcal{L}_{Yuk} = & - \sum_{\alpha=1}^3 [m_{d\alpha} \bar{d}_\alpha d_\alpha + m_{u\alpha} \bar{u}_\alpha u_\alpha] \\ & \times \left[1 + \frac{1}{v_1 \sqrt{2}} \sum_{r=1}^{2l-1} V_{1r}^{(0)} \hat{\chi}_r \right] \\ & - \sum_{i=1}^l \left\{ \frac{m_t}{v_1} \hat{\beta}_i \bar{t}_R \hat{\Phi}_i \sum_{\alpha=1}^3 V_{3\alpha} d_{\alpha L} + \text{h.c.} \right\} \\ & + \sum_{r=1}^{2l-1} \left\{ \frac{m_t}{v_1} \tilde{\beta}_r \bar{t}_R \hat{\chi}_r t_L + \text{h.c.} \right\}; \end{aligned} \quad (5.6)$$

$$\hat{\beta}_i = \sum_{j=2}^l \beta_j' V_{ji}^{(1)}, \quad (i = 1, \dots, l); \quad (5.7)$$

$$\begin{aligned} \tilde{\beta}_r &= \frac{1}{\sqrt{2}} \sum_{j=2}^l \beta_j' \left(V_{2j-2,r}^{(0)} + i V_{2j-1,r}^{(0)} \right), \\ & (r = 1, \dots, 2l-1). \end{aligned} \quad (5.8)$$

In order to compare with (7) of [17] let us just look at the ϕtb coupling implied by (5.6). We get

$$\begin{aligned} \mathcal{L}_{\phi tb} &= -\frac{m_t}{v_1} \sum_{i=1}^l \beta_i \bar{t}_R b_L \hat{\Phi}_i + \text{h.c.}, \\ \beta_i &= \hat{\beta}_i V_{33} = \sum_{j=2}^l \beta_j' V_{33} V_{ji}^{(1)}. \end{aligned} \quad (5.9)$$

It was shown in [17] that in models of the type considered here one gets nonzero effective CP-violating couplings (5.1) at the one-loop level, provided that

$$\text{Im} \beta_i \beta_j^* \neq 0 \quad \text{for some } i \neq j, \quad (5.10)$$

and the corresponding Higgs masses are non-degenerate. Clearly, in view of the large parameter space of the potential V (4.4), there is no reason why charged Higgs bosons should be mass-degenerate. Let us see if we can satisfy also (5.10). For $l = 2$ we get two charged physical Higgs fields and (5.9) gives

$$\begin{aligned}\beta_1 &= \beta'_2 V_{33} V_{21}^{(1)}, \\ \beta_2 &= \beta'_2 V_{33} V_{22}^{(1)}.\end{aligned}\quad (5.11)$$

Because the $V_{ij}^{(1)}$ can be chosen to be real without loss of generality (cf. Appendix C) we get

$$\text{Im } \beta_1 \beta_2^* = 0 \quad (5.12)$$

for arbitrary complex β'_2, V_{33} . Thus, no CP-violating effective couplings (5.1) can be induced in this model.

For $l = 3$, i.e., in a model with three charged physical Higgs fields the parameters β_i are

$$\begin{aligned}\beta_1 &= \beta'_2 V_{33} V_{21}^{(1)} + \beta'_3 V_{33} V_{31}^{(1)}, \\ \beta_2 &= \beta'_2 V_{33} V_{22}^{(1)} + \beta'_3 V_{33} V_{32}^{(1)}, \\ \beta_3 &= \beta'_2 V_{33} V_{23}^{(1)} + \beta'_3 V_{33} V_{33}^{(1)}.\end{aligned}\quad (5.13)$$

Because β'_2 and β'_3 are arbitrary complex numbers it is now easy to realize (5.10), e.g.,

$$\text{Im} \beta_1 \beta_2^* \neq 0. \quad (5.14)$$

Thus in models where we start with one charged $SU(2)$ Higgs singlet and at least three Higgs doublets we can in general have effective CP-violating, chirality-conserving couplings of the type (5.1) which remain nonzero for $m_b \rightarrow 0$. For the calculation of such couplings and a discussion of their magnitudes we refer to [17].

6 Conclusions

In this article we have analysed, for gauge theories with gauge group $G = SU(2) \times U(1)$ being spontaneously broken to the electromagnetic $U(1)_{em}$ group, some general properties of the coupling of scalar fields to the electroweak gauge bosons W^\pm, Z . We allowed the scalar fields to carry arbitrary representations of G . We found the following general results.

The structure of the Z -scalar-scalar coupling is determined by the charge matrix Q' and the matrix T'_3 of the third component of weak isospin, but both restricted to the space of physical scalars. We discussed the scalar field basis for which T'_3 is diagonal in the charge $q \neq 0$ sectors and has a standard form (3.28) in the $q = 0$ sector. This basis is in general not the mass eigenbasis and the rotation from the former to the latter led us to the introduction of orthogonal respectively unitary rotation matrices similar to the CKM matrix. We found that non-diagonal (complex) Z -charged scalar couplings in the mass eigenbasis require at least two (three) different eigenvalues of T'_3 in the corresponding charge sector.

Finally we investigated models with one charged Higgs singlet and any number l of Higgs doublets. In these models the Z -charged Higgs couplings in the mass eigenbasis can always be made real by a suitable rotation of fields. We considered then the coupling of these fields to quarks and gave examples of models where no flavour-changing neutral interactions at tree level occur. We showed that for $l \geq 3$ these models satisfy all requirements to have CP-violating and chirality-conserving effective $Zb\bar{b}G$ couplings (5.1) at the one-loop level as investigated in detail in [17]. Such couplings are then not suppressed by factors containing small quark masses. Thus further experimental search for such CP-violating couplings, which have been considered theoretically in [17]-[19] and experimentally in [20,21], should be quite interesting. Nonzero couplings of this kind would point to a rich structure in the scalar sector as shown in this article.

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Appendix A: Some properties of the $SU(2) \times U(1)$ representation carried by the Scalar Fields

Consider the real representation (2.15) of $G = SU(2) \times U(1)$ in the space of the real n -component Higgs fields $\chi(x) \in \mathbb{R}_n$ for each x (cf. (2.14)). We can trivially embed \mathbb{R}_n in the complex n -dimensional space \mathbb{C}_n and consider the orthogonal representation (2.15) of G as unitary representation of G in \mathbb{C}_n :

$$\begin{aligned}R(U) &: \mathbb{C}_n \rightarrow \mathbb{C}_n, \\ R^\dagger(U)R(U) &= \mathbb{1}.\end{aligned}\quad (A.1)$$

For the representation $U \rightarrow R(U)$, considered as unitary representation of G , all the standard results apply: It can be reduced completely. The hermitian generators are

$$\begin{aligned}T_a &= \frac{1}{i} \tilde{T}_a, \quad (a = 1, 2, 3); \\ Y &= \frac{1}{i} \tilde{Y}.\end{aligned}\quad (A.2)$$

The irreducible parts of the representation are characterised by (t, y) where $t \in \{0, 1/2, 1, \dots\}$ with $t(t+1)$ and y the eigenvalues of

$$T_a T_a = -\tilde{T}_a \tilde{T}_a, \quad \text{and } Y, \quad \text{respectively.} \quad (A.3)$$

Let y_1, \dots, y_n be the eigenvalues of Y . Clearly, the eigenvalues of

$$Y^2 = -\tilde{Y} \tilde{Y} = \tilde{Y}^T \tilde{Y} \quad (A.4)$$

are then y_j^2 ($j = 1, \dots, n$), as introduced after (2.20).

Now we want to discuss whether the representation $U \rightarrow R(U)$ in (A.1) is single- or multiple-valued. For the $SU(2)$ part of the group G there is no problem, since $SU(2)$ is singly connected. But the $U(1)$ part of G is multiply connected. The representation matrices of the group elements $U(0, \psi)$ in (2.1) corresponding to the $U(1)$ factor of G are:

$$\begin{aligned} U(0, \psi) &= e^{i\psi y_0} \rightarrow \exp(i\psi Y) \\ &= A^\dagger \text{diag} (e^{i\psi y_1}, \dots, e^{i\psi y_n}) A. \end{aligned} \quad (\text{A.5})$$

Here A is the matrix which diagonalises Y in \mathbb{C}_n . With the condition (2.21) we have

$$\begin{aligned} e^{i\psi y_j} |_{\psi=\pi y_0^{-1}} &= e^{i\psi y_j} |_{\psi=-\pi y_0^{-1}} \\ (j &= 1, \dots, n) \end{aligned} \quad (\text{A.6})$$

and thus the representation is single-valued: There is a single element $R(U)$ which corresponds to the element $U(0, \pm\pi) = (-\mathbb{1}) \in G$.

We will now show that the representation $U \rightarrow R(U)$, considered as a unitary representation in \mathbb{C}_n has the following property: If the irreducible representation (t, y) appears in the decomposition of the representation, then also $(t, -y)$ must occur. Furthermore the irreducible representations (t, y) and $(t, -y)$ must have the same multiplicity. To prove this, we note that due to (2.17) and (A.2) we have

$$\begin{aligned} (T_a T_a)^T &= (T_a T_a), \\ Y^T &= -Y. \end{aligned} \quad (\text{A.7})$$

Consider now the double resolvent:

$$\frac{1}{(\xi - T_a T_a)(\eta - Y)} = \sum_{t,y} \frac{\mathbb{P}(t, y)}{[\xi - t(t+1)](\eta - y)}, \quad (\text{A.8})$$

where ξ, η are arbitrary complex numbers and $\mathbb{P}(t, y)$ is the projector onto the subspace of \mathbb{C}_n carrying the representation (t, y) of G . (The irreducible representation (t, y) may occur with multiplicity one or higher). We have

$$\begin{aligned} \sum_{t,y} \mathbb{P}(t, y) &= \mathbb{1}, \\ \mathbb{P}(t, y)^\dagger &= \mathbb{P}(t, y). \end{aligned} \quad (\text{A.9})$$

From (A.7) we get

$$\left[\frac{1}{(\xi - T_a T_a)(\eta - Y)} \right]^T = \frac{1}{(\xi - T_a T_a)(\eta + Y)} \quad (\text{A.10})$$

which leads to

$$\sum_{t,y} \frac{\mathbb{P}(t, y)^T}{(\xi - t(t+1))(\eta - y)} = \sum_{t,y} \frac{\mathbb{P}(t, y)}{(\xi - t(t+1))(\eta + y)}. \quad (\text{A.11})$$

Comparing the location of the poles in (ξ, η) and the residues on the r.h.s. and l.h.s. of (A.11) we find that with

$(t(t+1), y)$ also $(t(t+1), -y)$ must be the location of a pole and

$$\mathbb{P}(t, -y) = \mathbb{P}(t, y)^T. \quad (\text{A.12})$$

This shows that the representations (t, y) and $(t, -y)$ occur with the same multiplicity, q.e.d.

Next we want to show the following theorem: If the vacuum expectation value v (2.24) satisfies

$$\mathbb{P}(t, y)v \neq 0, \quad (\text{A.13})$$

then y must be one of the eigenvalues of T_3 in the representation (t, y) :

$$y \in \{-t, -t+1, \dots, t\}. \quad (\text{A.14})$$

The proof is as follows: We have for the hermitian electric charge generator $Q = \frac{1}{i}\tilde{Q}$:

$$\begin{aligned} Q &= T_3 + Y = (T_3 + Y) \cdot \mathbb{1} \\ &= (T_3 + Y) \sum_{t,y} \mathbb{P}(t, y) \\ &= \sum_{t,y} (T_3 + y) \mathbb{P}(t, y). \end{aligned} \quad (\text{A.15})$$

From $Qv = 0$ (cf. (2.25)) and the fact that T_3 commutes with $\mathbb{P}(t, y)$, we get

$$\begin{aligned} Qv &= \sum_{t,y} (T_3 + y) \mathbb{P}(t, y)v \\ &= \sum_{t,y} \mathbb{P}(t, y)(T_3 + y) \mathbb{P}(t, y)v = 0, \\ \implies & \\ & \mathbb{P}(t, y)(T_3 + y) \mathbb{P}(t, y)v = 0, \\ \implies & \\ & (T_3 + y) \mathbb{P}(t, y)v = 0. \end{aligned} \quad (\text{A.16})$$

Thus, if $\mathbb{P}(t, y)v \neq 0$, then y is one of the eigenvalues of T_3 in the representation (t, y) of G , q.e.d.

From (A.12) we get for all (t, y) :

$$\begin{aligned} v^T \mathbb{P}(t, y)v &= v^T \mathbb{P}(t, -y)v \\ &= v^\dagger \mathbb{P}(t, y)v \geq 0. \end{aligned} \quad (\text{A.17})$$

Appendix B: Properties of gauge orbits of scalar fields

In this appendix we derive some properties of the gauge orbits of our general n component real scalar field χ in relation to the unitary gauge condition (2.38). In the following \mathbb{R}_{n-3} , as defined in (2.39), is the subspace of the real vectors χ satisfying the gauge condition (2.38).

Theorem 1: If $\chi \in \mathbb{R}_{n-3}$, then also $\chi_1 \in \mathbb{R}_{n-3}$ where

$$\chi_1 = R(U)\chi \quad (\text{B.1})$$

with U an arbitrary element of the electromagnetic subgroup $U_{em}(1) \subset G$.

Proof: For an arbitrary element $U \in U_{em}(1)$ we have with \tilde{Q} as in (2.19)

$$\begin{aligned} U &= \exp[i\tilde{\psi}(\frac{1}{2}\tau_3 + y_0)], \\ \chi_1 &= R(U)\chi \\ &= \exp(\tilde{\psi}\cdot\tilde{Q})\chi. \end{aligned} \quad (\text{B.2})$$

Then, using the commutation relations (2.20), we find immediately

$$v^T \tilde{T}_a \chi_1 = v^T \tilde{T}_a \exp(\tilde{\psi}\tilde{Q})\chi = 0, \quad (\text{B.3})$$

if $v^T \tilde{T}_a \chi = 0$ holds, i.e. if $\chi \in \mathbb{R}_{n-3}$. But (B.3) means that $\chi_1 \in \mathbb{R}_{n-3}$, q.e.d.

Theorem 2: There is no further subgroup $\tilde{G} \subset G$, $\tilde{G} \neq U_{em}(1)$, which leaves \mathbb{R}_{n-3} invariant. In other words: If for all $\chi \in \mathbb{R}_{n-3}$ also $R(U)\chi \in \mathbb{R}_{n-3}$, then $U \in U_{em}(1)$.

Proof (indirect): Assume, on the contrary, that \tilde{G} is a subgroup of G , $\tilde{G} \neq U_{em}(1)$, which leaves \mathbb{R}_{n-3} invariant. Then \tilde{G} contains at least one one-parameter subgroup $\bar{U}(1) \neq U_{em}(1)$. We must then have the following for the elements \bar{U} of $\bar{U}(1)$

$$\begin{aligned} R(\bar{U}) &= \exp(\tilde{\psi}\tilde{\tilde{Q}}), \\ (\bar{U} \in \bar{U}(1)), \end{aligned} \quad (\text{B.4})$$

where $\tilde{\tilde{Q}}$ is the matrix representing the generator of $\bar{U}(1)$:

$$\tilde{\tilde{Q}} = r_a \tilde{T}_a + r_4 \tilde{Q} \quad (\text{B.5})$$

with $r_a (a = 1, 2, 3)$ and r_4 real numbers and

$$(r_1, r_2, r_3) \neq (0, 0, 0), \quad (\text{B.6})$$

since $\bar{U}(1) \neq U_{em}(1)$ by assumption. Furthermore $\bar{U}(1)$ leaves \mathbb{R}_{n-3} invariant, which means:

$$v^T \tilde{T}_a R(\bar{U})\chi = v^T \tilde{T}_a \exp(\tilde{\psi}\tilde{\tilde{Q}})\chi = 0 \quad (\text{B.7})$$

for all $\chi \in \mathbb{R}_{n-3}$.

From (B.7) we find by differentiating with respect to $\tilde{\psi}$ at $\tilde{\psi} = 0$:

$$v^T \tilde{T}_a \tilde{\tilde{Q}}\chi = 0 \quad (\text{B.8})$$

for all $\chi \in \mathbb{R}_{n-3}$.

From the definition of \mathbb{R}_{n-3} in (2.39) we see that (B.8) can only hold if the vectors $v^T \tilde{T}_a \tilde{\tilde{Q}}$ are linearly dependent on $v^T \tilde{T}_b$ ($b = 1, 2, 3$):

$$v^T \tilde{T}_a \tilde{\tilde{Q}} = h_{ab} v^T \tilde{T}_b, \quad (h_{ab} \text{ real}). \quad (\text{B.9})$$

Multiplying by v from the right and using (2.27) we get

$$v^T \tilde{T}_a \tilde{\tilde{Q}}v = 0, \quad (\text{B.10})$$

$$\implies v^T \tilde{T}_a (r_b \tilde{T}_b + r_4 \tilde{Q})v = 0. \quad (\text{B.11})$$

Since $\tilde{Q}v = 0$ we get

$$v^T \tilde{T}_a (r_b \tilde{T}_b v) = 0. \quad (\text{B.12})$$

Multiplying with r_a ($a = 1, 2, 3$) and summing yields:

$$\begin{aligned} (v^T \tilde{T}_a^T r_a) \cdot (r_b \tilde{T}_b v) &= 0, \\ \implies \end{aligned} \quad (\text{B.13})$$

$$r_b \tilde{T}_b v = 0. \quad (\text{B.14})$$

Since $\tilde{T}_b v$ are linearly independent (cf. (2.26)), it follows that (B.14) can only hold if $(r_1, r_2, r_3) = (0, 0, 0)$. But this is a contradiction to (B.6). Thus, the assumption $\tilde{G} \neq U_{em}(1)$ is disproved and theorem 2 holds.

Let us now define rest classes in G with respect to $U_{em}(1)$:

$$U \sim U' \quad (\text{B.15})$$

if $UU'^{-1} \in U_{em}(1)$. Let \hat{G} be the set formed by these rest classes, i.e. the set of right cosets of $U_{em}(1)$. A parametrization of \hat{G} in a neighbourhood of the coset of the unit element of G is given by the elements of $SU(2) \subset G$ (cf. (2.1))

$$U(\varphi, 0) = \exp(i\frac{1}{2}\tau_a \varphi_a) \quad (\text{B.16})$$

with corresponding representation matrices

$$R(U(\varphi, 0)) = \exp(\varphi_b \tilde{T}_b). \quad (\text{B.17})$$

In general the elements $U \in G$ which transform a given vector $\chi \in \mathbb{R}_n$ into a vector $\chi_1 \in \mathbb{R}_{n-3}$

$$R(U)\chi = \chi_1 \in \mathbb{R}_{n-3} \quad (\text{B.18})$$

form isolated points in the coset space \hat{G} .

This can be shown as follows. Let χ be an arbitrary vector from \mathbb{R}_n and let $U_1 \in G$ be a transformation such that

$$R(U_1)\chi = \chi_1 \in \mathbb{R}_{n-3}. \quad (\text{B.19})$$

A suitable parametrization for the cosets in a neighbourhood of the coset of U_1 is given by the following elements of G :

$$U(\varphi, 0) \cdot U_1. \quad (\text{B.20})$$

We have to study the system of equations

$$\begin{aligned} h_a(\varphi) &:= v^T \tilde{T}_a R(U(\varphi, 0))R(U_1)\chi \\ &= v^T \tilde{T}_a R(U(\varphi, 0)\chi_1 \\ &= 0 \end{aligned} \quad (\text{B.21})$$

near $\varphi = 0$. We have

$$\left. \frac{\partial}{\partial \varphi_b} h_a(\varphi) \right|_{\varphi=0} = v^T \tilde{T}_a \tilde{T}_b \chi_1. \quad (\text{B.22})$$

The point $\varphi = 0$ is an isolated solution of (B.21) if

$$\det(v^T \tilde{T}_a \tilde{T}_b \chi_1) \neq 0. \quad (\text{B.23})$$

If this determinant equals zero, $\varphi = 0$ need not be an isolated solution.

Let us define the set

$$M' = \{\chi_1 | \chi_1 \in \mathbb{R}_{n-3}, \det(v^T \tilde{T}_a \tilde{T}_b \chi_1) = 0\}. \quad (\text{B.24})$$

The determinant being zero represents one algebraic equation for the vectors $\chi_1 \in \mathbb{R}_{n-3}$. Thus the dimension of M' can be at most $n - 4$.

Theorem 3: There exists a neighbourhood of the vacuum expectation value $\chi_1 = v$ in \mathbb{R}_{n-3} which has no point in common with M' .

Proof: Since $T_a v$ ($a = 1, 2, 3$) are linearly independent (cf. (2.26)) we have

$$\det(v^T \tilde{T}_a \tilde{T}_b v) \neq 0. \quad (\text{B.25})$$

By continuity we have then

$$\det(v^T \tilde{T}_a \tilde{T}_b \chi_1) \neq 0 \quad (\text{B.26})$$

for χ_1 in a suitable neighbourhood of v in \mathbb{R}_{n-3} , q.e.d.

Theorem 4: The manifold M' is invariant under the action of the electromagnetic group $U_{em}(1)$:

$$\chi_2 = R(U)\chi_1 \in M' \quad \text{if} \quad \chi_1 \in M' \quad \text{and} \quad U \in U_{em}(1). \quad (\text{B.27})$$

Proof: Indeed, for $U \in U_{em}(1)$ we have with the notation according to (B.2):

$$\begin{aligned} R^T(U) \tilde{T}_a R(U) &= \exp(\tilde{\psi} \tilde{Q}^T) \tilde{T}_a \exp(\tilde{\psi} \tilde{Q}) \\ &= \exp(-\tilde{\psi} \tilde{T}_3) \tilde{T}_a \exp(\tilde{\psi} \tilde{T}_3) \\ &= D_{ab}(U') \tilde{T}_b, \end{aligned} \quad (\text{B.28})$$

where $U' = \exp(i\tilde{\psi}\tau_3/2)$ and $(D_{ab}(U'))$ is the matrix of the adjoint representation of $SU(2)$. We get then:

$$\begin{aligned} v^T \tilde{T}_a \tilde{T}_b \chi_2 &= v^T R(U) R^T(U) \tilde{T}_a R(U) R^T(U) \tilde{T}_b R(U) \chi_1 \\ &= D_{aa'}(U') D_{bb'}(U') v^T \tilde{T}_{a'} \tilde{T}_{b'} \chi_1, \\ \implies \\ \det(v^T \tilde{T}_a \tilde{T}_b \chi_2) &= \det(v^T \tilde{T}_{a'} \tilde{T}_{b'} \chi_1), \end{aligned} \quad (\text{B.29})$$

q.e.d.

Consider next the manifold $M \in \mathbb{R}_n$ of those Higgs fields χ whose gauge orbits intersect \mathbb{R}_{n-3} in M' :

$$M = \{\chi | \chi \in \mathbb{R}_n, \text{ such that there exists } U_1 \in G \text{ with } R(U_1)\chi = \chi_1 \in M'\}. \quad (\text{B.30})$$

We have then

$$\chi = R^{-1}(U_1)\chi_1, \quad \chi_1 \in M'. \quad (\text{B.31})$$

Since M' is invariant under the action of $U_{em}(1)$ it is sufficient to choose for U_1 in (B.31) only one representative of each right coset of $U_{em}(1)$. This means that the parameters needed to describe the manifold M are those of M' (at most $n-4$) and the 3 parameters (at most) of the coset

space \hat{G} . Thus M has at most dimension $n-4+3 = n-1$ and is a set of measure zero in \mathbb{R}_n .

We summarize these findings as follows.

Theorem 5: The elements $U \in G$ which transform a given Higgs field $\chi \in \mathbb{R}_n$ (at a given space-time point) into a vector χ_1 satisfying the gauge condition (2.38) belong for general χ to isolated points in the coset space \hat{G} . The vectors $\chi \in \mathbb{R}_n$ where this is not the case form a manifold M of dimension $\leq n-1$ in \mathbb{R}_n and thus a set of measure zero.

Finally, let us discuss the question of multiple intersections of the gauge orbit of a vector χ with \mathbb{R}_{n-3} (2.39). For given $\chi \in \mathbb{R}_n$ we have

$$\chi_1 = R(U_0)\chi \in \mathbb{R}_{n-3},$$

where $U_0 \in G$ is the transformation that maximises the function $f(U)$ (2.40). It is clear that also the element $U'_0 \in G$ which minimises $f(U)$:

$$f(U) \geq f(U'_0) \text{ for all } U \in G$$

leads to an intersection of the gauge orbit of a vector χ with \mathbb{R}_{n-3} :

$$\chi_2 := R(U'_0)\chi \in \mathbb{R}_{n-3}.$$

The same holds true for all stationary points of $f(U)$. Thus, in general, the gauge orbit of χ will have multiple intersections with \mathbb{R}_{n-3} . Consequently the gauge condition (2.38) can and must be sharpened by restricting χ to a region in \mathbb{R}_{n-3} where the gauge orbits have single intersections only. A suitable restriction is to the region in \mathbb{R}_{n-3} defined by the absolute maxima of the functions $f(U)$:

$$R'_{n-3} = \{\chi | \chi \in \mathbb{R}_{n-3}, v^T R(U)\chi \leq v^T \chi \text{ for all } U \in G\} \quad (\text{B.32})$$

Here we assume that the absolute maxima of $v^T R(U)\chi$ have no degeneracy (except for a set of measure zero).

In the SM with one Higgs doublet the restriction of the form (B.32) is, of course, well known. Taking it into account in the path integral quantisation by the method of Fadeev and Popov [22] one finds that even in the unitary gauge ghost fields are required. This was first demonstrated in the canonical quantisation procedure by Weinberg [5].

Appendix C: Properties of the matrix $V^{(1)}$

In this appendix we show that the matrix $V^{(1)}$ of (4.16) can always be chosen to be real. According to (3.54) we are free to make the transformations

$$V^{(1)} \rightarrow U_1^\dagger V^{(1)} U_2 \quad (\text{C.1})$$

where

$$U_2 = \text{diag}(e^{i\psi_1}, \dots, e^{i\psi_1}). \quad (\text{C.2})$$

and U_1 is a unitary $l \times l$ matrix commuting with $T_3^{(1)}$ (4.15). Thus U_1 must have the form

$$U_1 = \left(\begin{array}{c|c} e^{i\varphi_1} & 0 \\ \hline 0 & U'_1 \end{array} \right) \quad (\text{C.3})$$

with U'_1 being a unitary $(l-1) \times (l-1)$ matrix.

With a suitable choice of U_1 and U_2 in (C.1) we can achieve

$$\begin{aligned} V_{11}^{(1)}, V_{12}^{(1)}, \dots, V_{1l}^{(1)} & \text{ real,} \\ V_{21}^{(1)} & \text{ real, } V_{31}^{(1)} = \dots = V_{l1}^{(1)} = 0. \end{aligned} \quad (\text{C.4})$$

Two cases can be distinguished: (i) $V_{21}^{(1)} = 0$. Then applying (C.1) we can immediately bring $V^{(1)}$ to the real form:

$$V^{(1)} = \left(\begin{array}{c|c} V_{11}^{(1)} & 0 \\ \hline 0 & \mathbb{1}_{l-1} \end{array} \right). \quad (\text{C.5})$$

(ii) $V_{21}^{(1)} \neq 0$. Then the unitarity relations for $V^{(1)}$ require

$$V_{22}^{(1)}, \dots, V_{2l}^{(1)} \text{ real.}$$

By a suitable choice of U_1, U_2 in (C.1) we can then achieve

$$V_{32}^{(1)} \text{ real, } V_{42}^{(1)} = \dots = V_{l2}^{(1)} = 0 \quad (\text{C.6})$$

and repeat the above reasoning (i) and (ii) with $V_{32}^{(1)}$ playing the role of $V_{21}^{(1)}$. In this way we see that indeed $V^{(1)}$ can be made real by a transformation (C.1).

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